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Time-Adaptive Sampling of a Chemical Hazard Area

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Overview

- Background
- Problem statement and assumptions
- Methodology
- Illustrations
- Conclusions
- Future Work

Background

- Chemical and biological WMDs are a current threat to the United States
  - 2001 U.S.: anthrax attacks
  - 1998 Iraq: "cocktail" of weapons killed $5,000^{+}$
  - 1995 Tokyo: sarin nerve gas, killed 12, injured thousands
- Terrorist groups are willing to use asymmetric measures
  - Easy manufacturing, storing, and transportation appeal to terrorists

Problem Dynamics

- A chemical agent weapon is released over a fixed operational site
  - Entire site enters the highest level of MOPP
  - Contamination from secondary vapors is the main concern

- Reduce mission oriented protective posture (MOPP)
  - MOPP is cumbersome
  - High levels of MOPP can reduce work efficiency

Problem Dynamics

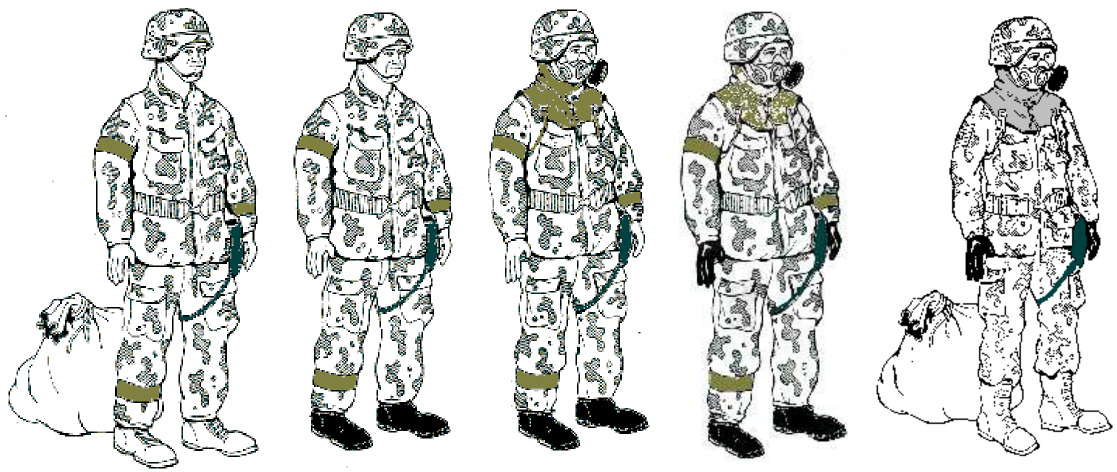


Figure 1. Mission-oriented protective postures.

- MOPP 0, MOPP 1, MOPP 2, MOPP 3, MOPP 4, MOPP Alpha
- Progressively add gear for increased safety

Ref:AFVA 32-4012, http://www.fas.org/nuke/guide/usa/doctrine/usaf/32401200.pdf, 1 March 2005.

Problem Statement

- Develop an optimal sampling strategy
  - Route a search crew
    - Reach as many locations as possible (to identify maximum number of areas below the vapor concentration threshold)
    - Time constraint
- Provide a framework for future work
  - Using sensor data
  - Predicting future hazard areas

Model Assumptions

- Rectangular region with a finite number of “critical” areas
- Single crew that samples vapor concentrations
- Static, deterministic, and symmetric travel times
- Travel at constant velocity with zero delays
- Fixed amount of time allotted for the search

Model Assumptions

- Chemical agent/characteristics are known
- Only one instrument reading is required, consuming a fixed amount of time
- Known fixed threshold indicating contamination/no contamination
- Secondary vapor concentrations evolve spatially

Optimization Model

- Model the site and its critical areas as a network

- Develop a technique for optimally searching the site

- Desired outcome: Identify areas where secondary vapor levels have decreased (below the fixed vapor concentration level v^*) so MOPP can be safely reduced at those locations

Network Model

Consider the following notional site

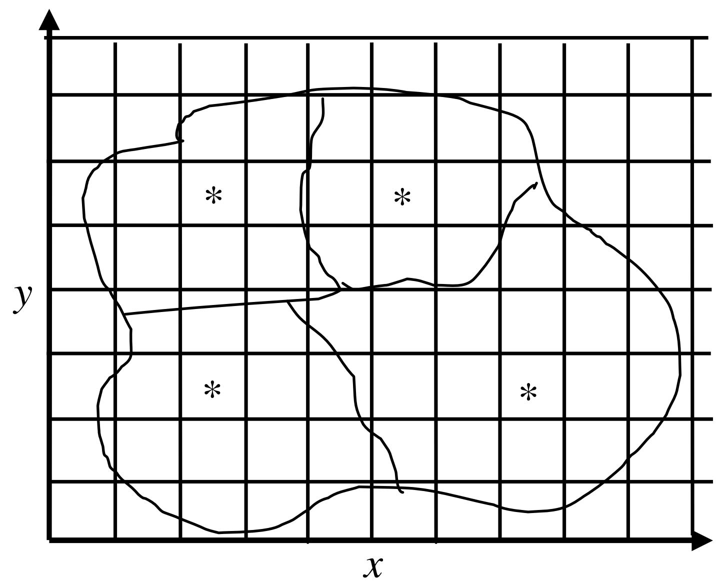


Figure 2. Graphical depiction of areas on an installation.

Network Model

Definitions:

- $G = (\mathcal{N}, \mathcal{A})$ describes the graph with:
  - $\mathcal{N} \equiv \{1, 2, ..., N\}$, where N is the number of critical areas
  - $\mathcal{A} \equiv$ set of arcs (i, j) for $i, j \in \mathcal{N}$

Network Model

- $\mathcal{N}_i \equiv$ set of nodes adjacent to node i

- $t_{i,j} \equiv$ constant time required to travel from node i to node j
  - $t_{i,j} > 0, \forall (i,j) \in \mathcal{A}$
  - $t_{i,j} = t_{j,i}$

- $v_j(t) \equiv$ nonnegative vapor concentration at node $j \in \mathcal{N}$ at time t

- $r_j(t) \equiv$ binary reward received from searching node j at time t

Network Model

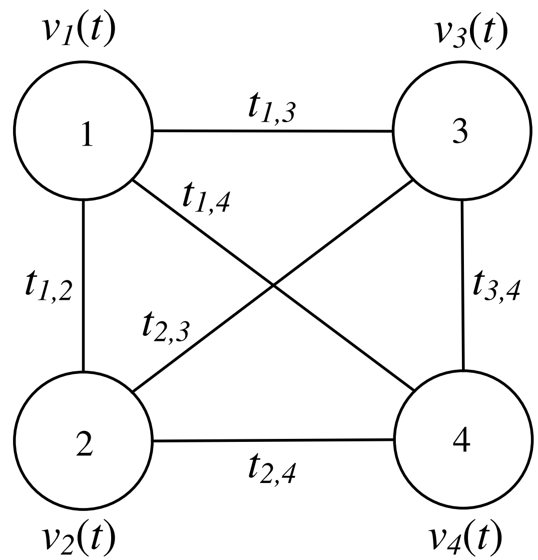


Figure 3. Example of the network representation for a 4-node site.

- $\mathcal{N} = \{1, 2, 3, 4\}$
- $\mathcal{A} = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4), (2,1), (3,1), (4,1), (3,2), (4,2), (4,3)\}$

Well-known Network Models

- Shortest Path Problem (SPP)
  - Path from source to sink
  - Not all nodes must be reached
- Knapsack Problem
  - Maximize a value with a constraint on the resource
  - Order does not matter

Well-known Network Models

- Travelling Salesperson Problem (TSP)
  - Minimize tour length
  - Must reach every city
  - Start and end at the origin

Methodology

We consider four distinct cases:

- Static and deterministic vapor concentrations
- Static and stochastic vapor concentrations
- Dynamic and deterministic vapor concentrations
- **Dynamic and stochastic vapor concentrations**

Dynamic and Deterministic

- Deterministic:
  - Assume vapor level concentration at each node can be calculated deterministically

- Dynamic:
  - Vapor levels depend on time, $v_j(t)$, for all $j \in \mathcal{N}, t \geq 0$

Dynamic and Deterministic

Objective: Maximize reward:

$$\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j(t) x_{i,j}$$

- Time constraint
- Backtracking *is* allowed
- Vapor concentrations are dynamic $\Rightarrow$ rewards are dynamic
- Possibly not all nodes will be reached

Algorithm

Initialization:

$\mathcal{N} = \{1, 2, ..., N\}$; $\mathcal{N}_i = \{j : i \to j\}$;

$\mathcal{R} = \emptyset$; $\psi = \emptyset$;

$t \leftarrow t_0$;

$i = 1$;

Calculate current vapor level at node i, $v_i(t)$

If $v_i(t) < v^*$

$r_i(t) \leftarrow 1$;

$\mathcal{R} \leftarrow \{i\}$; $\psi \leftarrow \psi \cup \{i\}$;

Else

$r_i(t) \leftarrow 0$;

$\psi \leftarrow \psi \cup \{i\}$;

End

Algorithm

Step 1

Calculate $v_j(t + t_{i,j})\ \forall j \ \in \mathcal{N}_i$

If $v_j(t + t_{i,j}) < v^*$

$r_j(t + t_{i,j}) \leftarrow 1;$

Else

$r_j(t + t_{i,j}) \leftarrow 0;$

End

Algorithm

Step 2

For each $j \in \mathcal{N}$ such that $r_j = 1$

Choose j such that $v_j(t + t_{i,j}) = \arg\min_{j \in \mathcal{N}_i}\{v^* - v_j(t + t_{i,j})\}$

$\mathcal{R} \leftarrow \mathcal{R} \cup \{j\}$;

$\psi \leftarrow \psi \cup \{j\}$;

End

If $r_j(t + t_{i,j}) = 0 \ \forall \ j \in \mathcal{N}_i$

Choose j such that $t_{i,j} = \min_j\{t_{i,j}\} \forall j \in \mathcal{N}_i$

$\psi \leftarrow \psi \cup \{j\}$;

End

$t \leftarrow t + t_{i,j}$;

Algorithm

Step 3

If $t \geq T$

STOP

Else

$i \leftarrow j;$

Return to Step 1

End

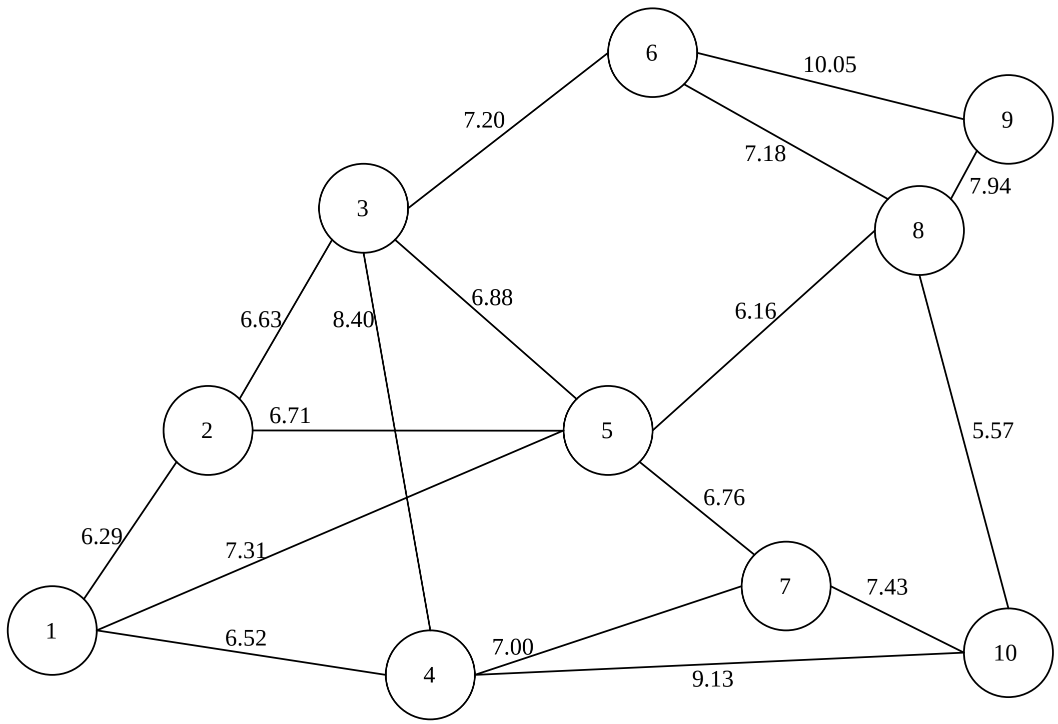
Dynamic/Deterministic

Result: iterative process yields a time-adaptive policy

- Future decisions depend on arrival times at nodes
- Vapor concentrations (rewards) drive the solution

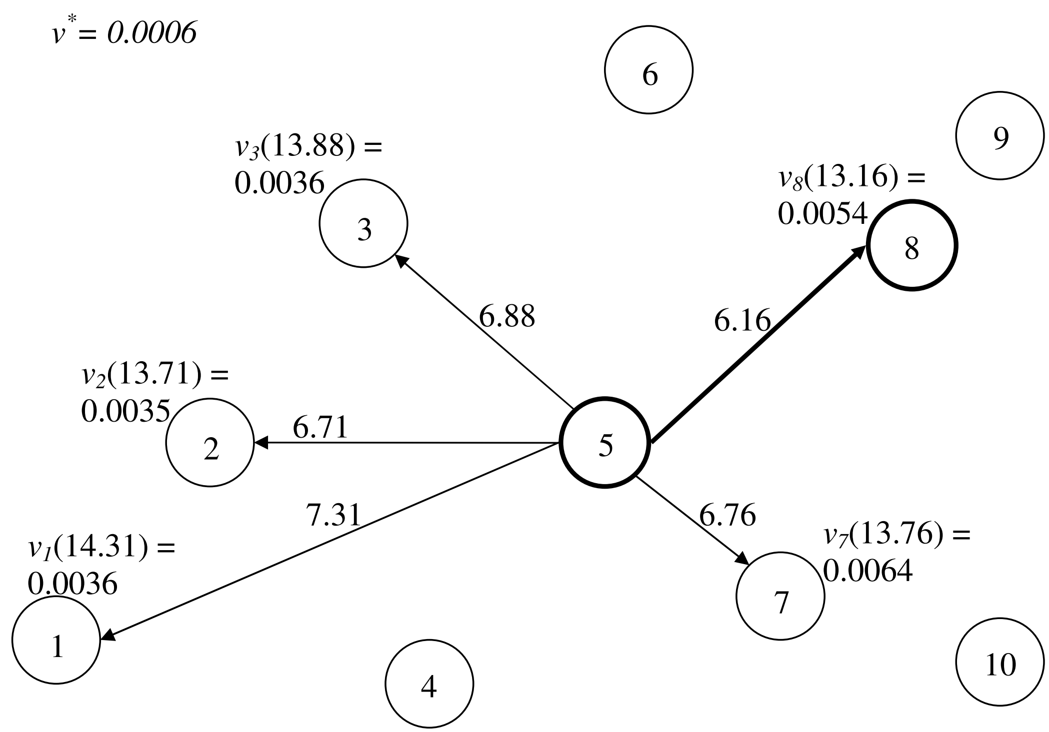
Dynamic/Deterministic

Network Configuration

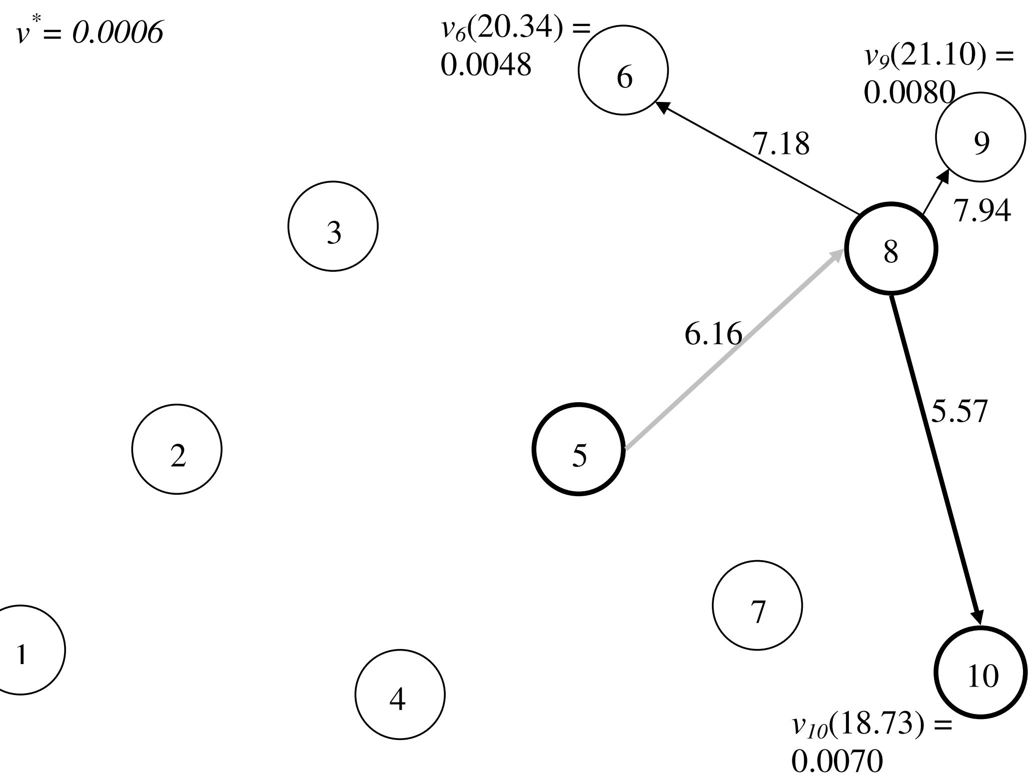


Dynamic/Deterministic

Example: Iteration 1: $t_0 = 7$

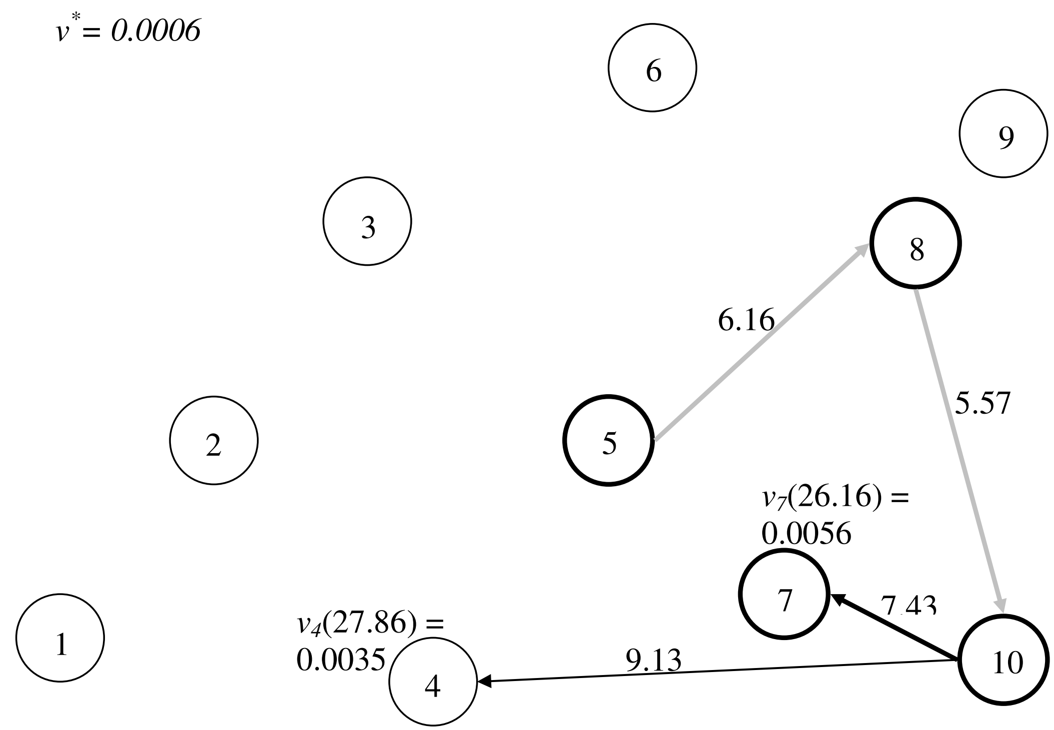


Iteration 2:


v* = 0.0006
v6(20.34) =
0.0048
6
v9(21.10) =
0.0080
9
7.18
7.94
3
8
6.16
5.57
2
5
7
1
4
10
v10(18.73) =
0.0070

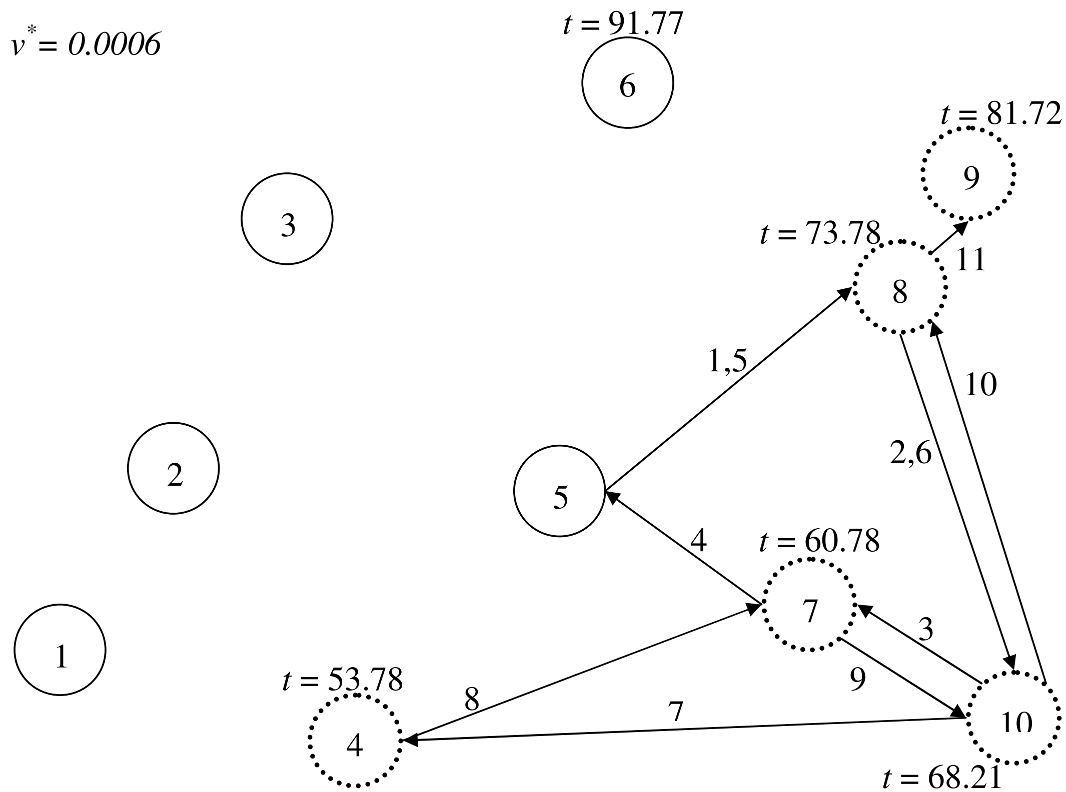
Dynamic/Deterministic

Iteration 3:



Dynamic/Deterministic

Final Solution:



Dynamic/Deterministic

Table 1. Vapor concentrations and rewards for nodes in ψ.

Node	t (min)	$v_j(t)$	$r_j(t)$
5	7.00	0.000650	0
8	13.16	**0.005400**	0
10	18.73	**0.007000**	0
7	26.16	**0.005600**	0
5	32.92	0.002800	0
8	39.08	0.002200	0
10	44.65	0.001400	0
4	53.78	0.000410	1
7	60.78	**0.000330**	1
10	68.21	**0.000190**	1
8	73.78	**0.000110**	1
9	81.72	0.000059	1

Dynamic/Deterministic

Table 2. Vapor concentrations and rewards at termination.

Node	$v_j(81.72)$	$r_j(\tau^*)$
1	0.000023	1
2	0.000025	1
3	0.000030	1
4	0.000034	1
5	0.000040	1
6	0.000042	1
7	0.000052	1
8	0.000054	1
9	0.000059	1
10	0.000057	1

Dynamic and Stochastic

- Time-variant probability distribution for each node (e.g., $V_j(t) \sim \exp(\mu_j(t))$ for all $j \in \mathcal{N}$)

- Objective: Maximize reward - The number of areas searched where the vapor concentration has *most likely* decreased below v^*

  – Reward is dynamic and computed from the expected value

  – If $E[V_j(t)] < v^*$, $r_j(t) = 1$, otherwise $r_j(t) = 0$.

Algorithm

Initialization:

$\mathcal{N} = \{1, 2, ..., N\}$; $\mathcal{N}_i = \{j : i \to j\}$;

$\mathcal{R} = \emptyset$; $\psi = \emptyset$;

$t \leftarrow t_0$;

$i = 1$;

Obtain realization of vapor level $v_i(t)$

If $v_i(t) < v^*$

$r_i(t) \leftarrow 1$;

$\mathcal{R} \leftarrow \mathcal{R} \cup \{i\}$; $\psi \leftarrow \psi \cup \{i\}$;

Else

$r_i(t) \leftarrow 0$;

$\psi \leftarrow \psi \cup \{i\}$;

End

Algorithm

Step 1

Calculate $\pi_j(t + t_{i,j}) \equiv P\{V_j(t + t_{i,j}) < v^*\} \ \forall j \ \in \mathcal{N}_i$

Step 2

Choose j such that $\pi_j(t) = \max_{j \in \mathcal{N}_i} P\{V_j(t + t_{i,j}) < v^*\}$

Obtain instrument reading at this node.

If $v_j(t + t_{i,j}) < v^*$

$r_j(t) \leftarrow 1$; $\mathcal{R} \leftarrow \mathcal{R} \cup \{j\}$; $\psi \leftarrow \psi \cup \{j\}$;

$t \leftarrow t + t_{i,j}$;

Else

$r_j(t) \leftarrow 0$;

$\psi \leftarrow \psi \cup \{j\}$;

$t \leftarrow t + t_{i,j}$;

End

Algorithm

Step 3

If $t \geq T$

STOP

Else

$i \leftarrow j$;

Return to Step 1

End

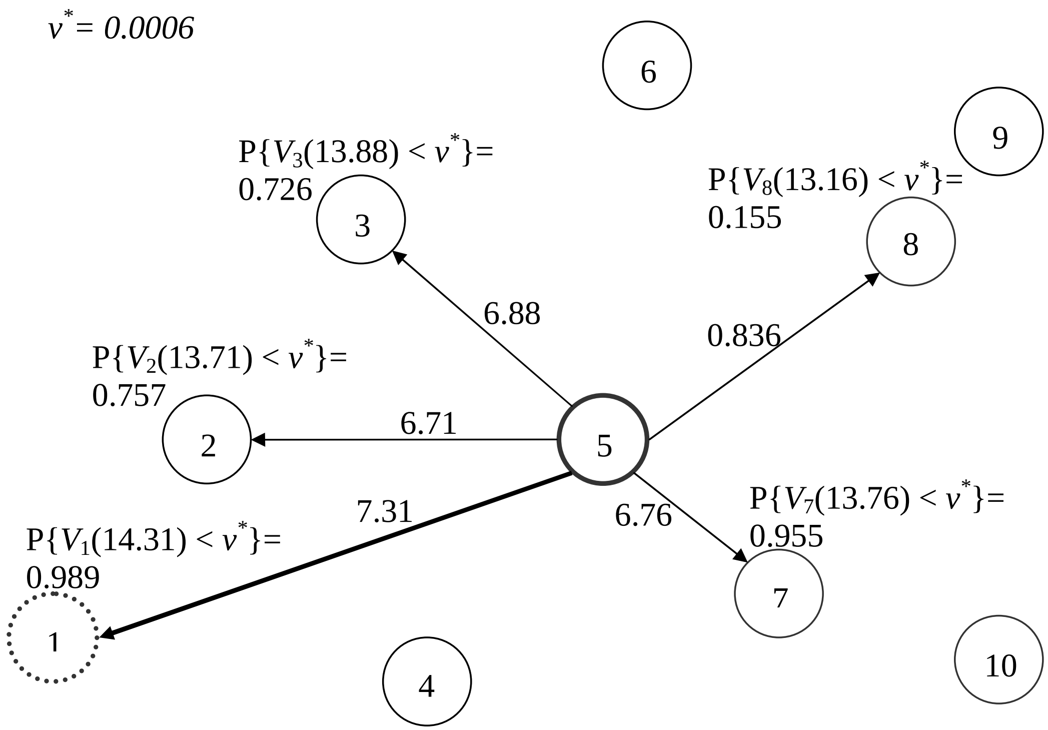
Dynamic/Stochastic

Result: iterative process yields a time-adaptive policy

- Future decisions depend on arrival times
- *Probability* a vapor concentration is below the threshold v^* drives the solution

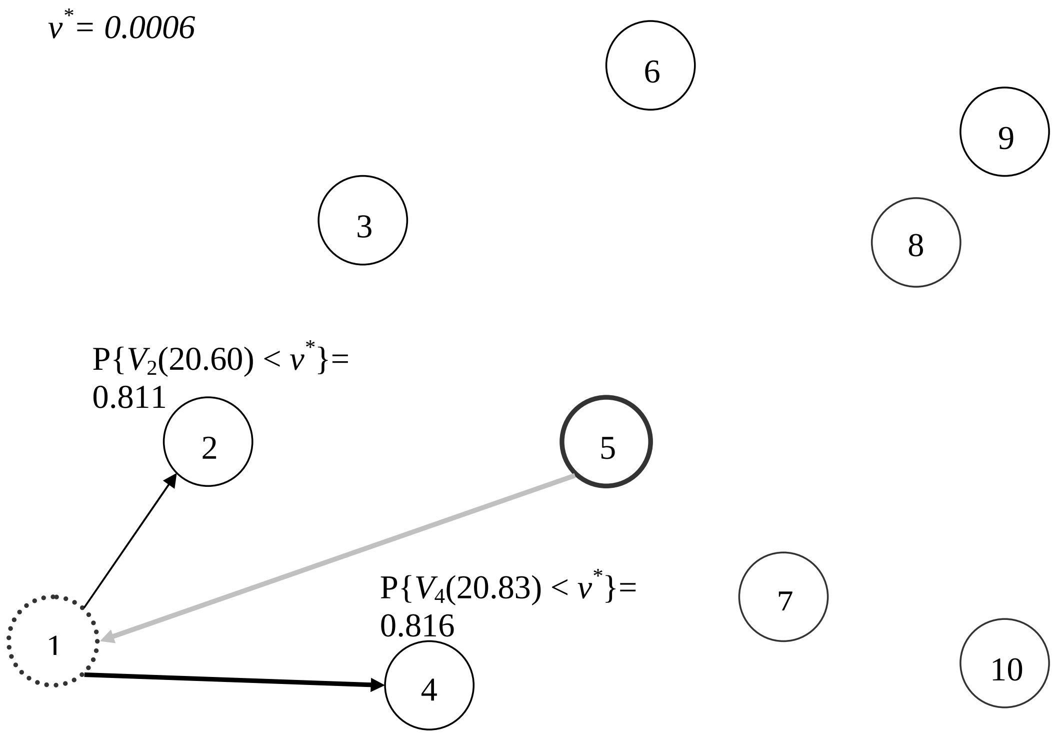
Dynamic/Stochastic

Example: Iteration 1:



Dynamic/Stochastic

Iteration 2:



Dynamic/Stochastic

Table 3: Vapor concentrations for nodes in ψ ($v^* = 6.0 \times 10^{-4}$).

Node	t (min)	$v_j(t)$	$r_j(t)$
5	7	0.0011	0
1	14.31	0.000012	1
4	20.83	0.000106	1
10	29.96	0.000172	1
7	37.39	0.000103	1
5	44.15	0.000113	1
1	51.46	0.000172	1
4	57.98	0.000179	1
10	67.11	0.000609	1
7	74.54	0.000693	1
5	81.3	0.000803	1
1	88.61	0.000001	1

Summary/Conclusions

- Ignoring dynamics may lead to under- or over-estimation of the number of safe areas (in these examples)

- Spatiotemporal characteristics are critical in developing the sampling strategy

- Want to minimize Type II error (i.e., accept H_0 that area is safe given it is not)

- Data was assumed to exist for illustrative purposes, however...

- Real problem presents significant data requirements

Future Work

- Relax assumptions
  - Consider non-deterministic travel times
  - Multiple search crews
  - Estimate probability distributions
- Incorporate real-time information
  - Real-time concentration readings from sensors
  - Road closures/openings
  - Weather changes (e.g., wind velocity, temperature, humidity, etc.)

Questions?

Backups

Backup: A-D Equation

The following parameters must be known to employ advection diffusion equation to compute v_j:

$x, y, z \equiv$ coordinates in the direction of the mean wind, horizontal cross-wind, and upwards vertical direction.

$k_x, k_y, k_z \equiv$ eddy diffusivities in m^{2}sec^{-1}

$q \equiv$ the total mass release in kg

$h \equiv$ instantaneous gas release height above the ground in m

$u \equiv$ wind velocity in m/sec

Backup: A-D Equation

$$v_j = \frac{q}{8\pi^{\frac{3}{2}}(k_x k_y k_z)^{1/2} t_0^{3/2}} \exp\left[-\frac{(x-ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0}\right] \times \left(\exp\left[-\frac{(z-h)^2}{4k_z t_0}\right] + \exp\left[-\frac{(z+h)^2}{4k_z t_0}\right]\right). \quad (1)$$

Ref: Kathirgamanathan, P., McKibbin, R., and R.I. McLachlan (2003). Source release-rate estimation of atmospheric pollution from a non-steady point source - Part 1: Source at a known location. *Res. Lett. Inf. Math. Sci.*, **5**, 71-84.

Backup: A-D Equation

Equation 1 can be simplified to

$$v_j = \frac{q}{8\pi^{\frac{3}{2}}(k_x k_y k_z)^{1/2} t_0^{3/2}} \exp\left[-\frac{(x-ut_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0}\right] \times \left(2\exp\left[-\frac{h^2}{4k_z t_0}\right]\right), \quad (2)$$

since $z = 0$ for our numerical illustrations.

Backup: Initial Rate Parameters

Table 6. Rate parameters chosen for the exponential distributions used for example 2.

Node	μ_j	$E[V_j](\times 10^{-4})$	r_j
1	6666.67	1.50	1
2	1538.46	6.50	0
3	1322.75	7.56	0
4	1574.80	6.35	0
5	1754.39	5.70	1
6	10000.00	1.00	1
7	4347.83	2.30	1
8	2222.22	4.50	1
9	909.09	11.00	0
10	7692.31	1.30	1

Case 1: Static and Deterministic

- Deterministic:
  - Assume vapor level concentration at each node is calculated via a deterministic formula immediately after the attack
- Static:
  - Assume for each $j \in \mathcal{N}$, v_j does not evolve over time

Case 1: Static/Deterministic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward (i.e., maximum number of areas not requiring protective gear).

- Time constraint implies it is possible that not all areas will be sampled

- No backtracking unless necessary (i.e., there is no reward for returning to an area)

- No subtours ($\mathcal{S} \subset \mathcal{N} \equiv$ set of all possible subtours)

Case 1: Static/Deterministic

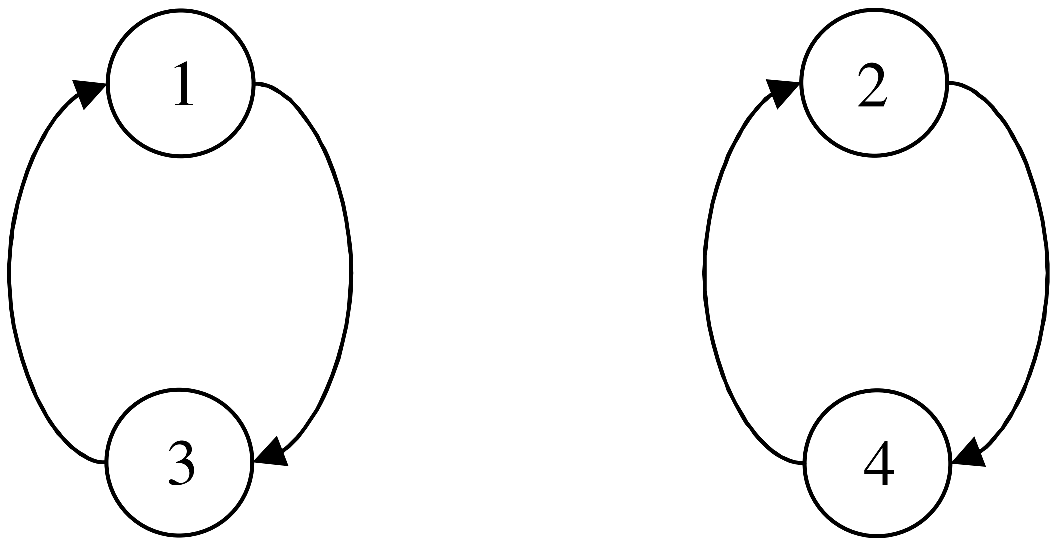


Figure 4. Example of subtour in a 4-node site.

Case 1: Static/Deterministic

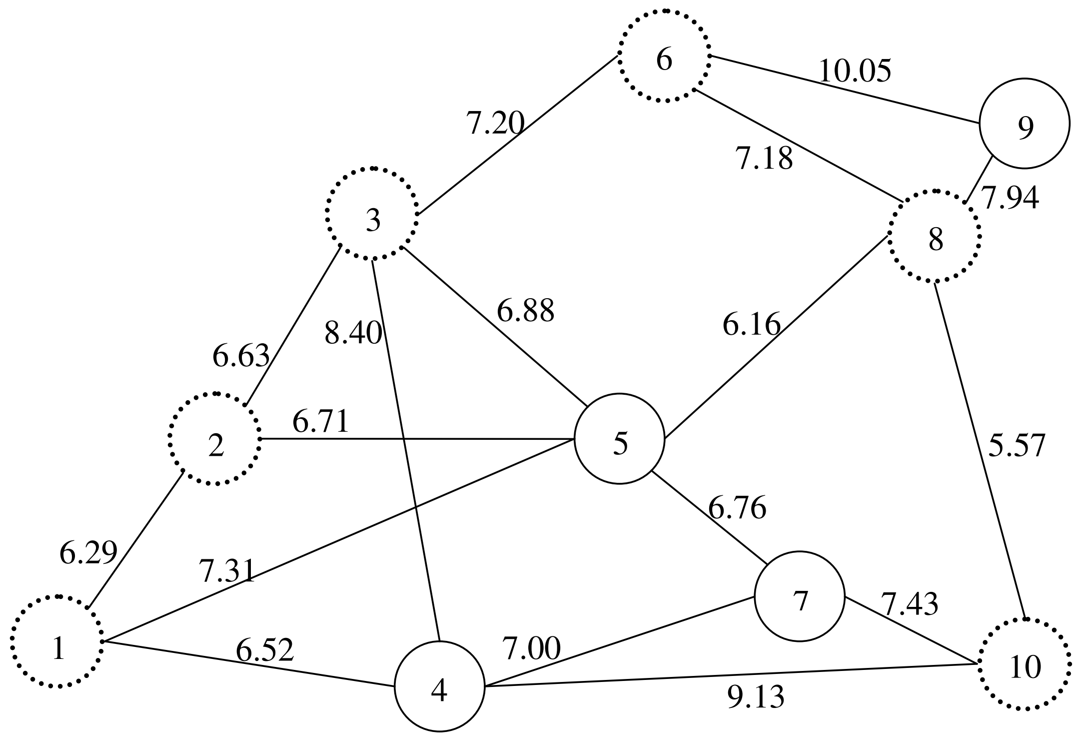
$$\max \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} r_j x_{i,j} (opt), \quad \min \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} t_{i,j} x_{i,j}$$

subject to

$$\begin{aligned}
\sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} t_{i,j} x_{i,j} &< T \\
\sum_{j\in\mathcal{N}} x_{s,j} &= 1 \text{ for } s \in \mathcal{N} \\
\sum_{i\in\mathcal{N}} x_{i,j} &\leq 1 \text{ for } j = 1, ..., N; j \neq i \\
\sum_{j\in\mathcal{N}} x_{i,j} &\leq 1 \text{ for } i = 1, ..., N; i \neq j \\
x_{i,j} + x_{j,i} &\leq 1 \text{ for all } (i,j) \in \mathcal{A} \\
\sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} x_{i,j} &\leq |\mathcal{S}| - 1 \text{ for } \mathcal{S} \in \mathcal{N}, 2 \leq |\mathcal{S}| \leq N - 1 \\
x_{i,j} &\in \{0, 1\}
\end{aligned}$$

Case 1: Static/Deterministic

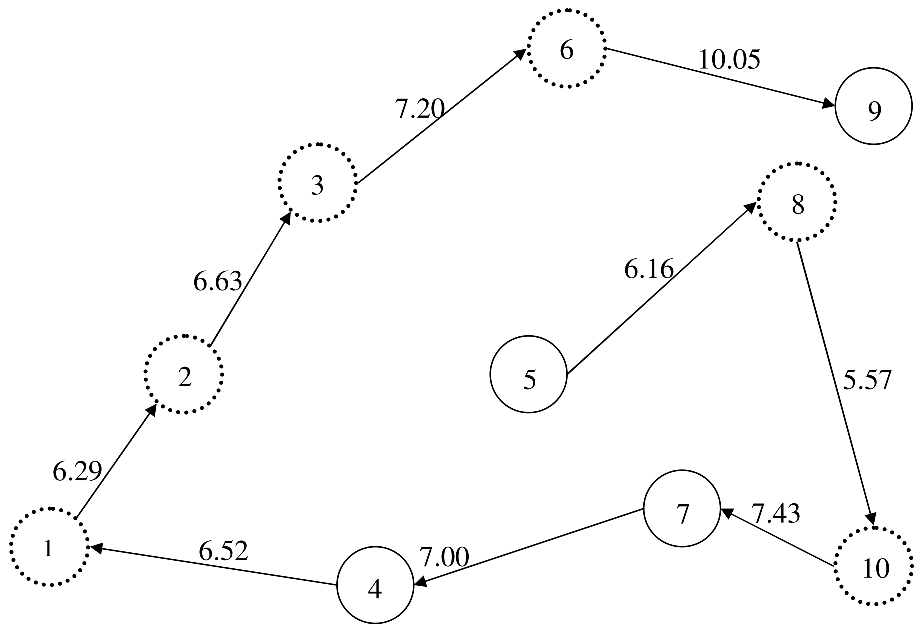
Example 1: Consider the following 10-node network



$r_j = 1$ for $j = 1, 2, 3, 6, 8, 10$; $r_j = 0$ for $j = 4, 5, 7, 9$

Case 1: Static/Deterministic

Solution:



- Total time of search: $\tau^* = 62.85$ minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
  Total reward: $r^* = 6$ from nodes 1, 2, 3, 6, 8, 10

Case 2: Static and Stochastic

- Stochastic: Assume vapor level concentration at each node is a random variable V_j, for all $j \in \mathcal{N}$, with an associated probability distribution

- Static: $P\{V_j \leq v^*\}$ does not change with time, nor does $E[V_j]\ \forall j \in \mathcal{N}$

Case 2: Static/Stochastic

Objective: Minimize time required to reach as many areas as possible to obtain the maximum reward

- Same formulation as Case 1

- Rewards are found from expected vapor concentrations
  - E.g., $V_j \sim \exp(\mu_j)$ for all $j \in \mathcal{N}$
  - $E[V_j] = \frac{1}{\mu_j}$
  - If $E[V_j] < v^*$, $r_j = 1$, otherwise $r_j = 0$.

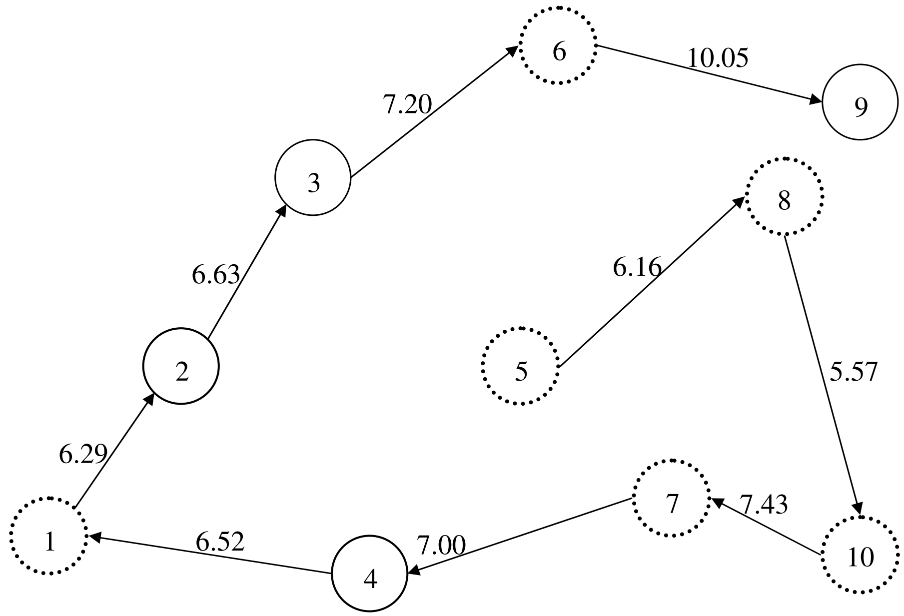
Case 2: Static/Stochastic

$$\max \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} r_j x_{i,j} (opt), \quad \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j}$$

subject to

$$\begin{aligned}
\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} t_{i,j} x_{i,j} &< T \\
\sum_{j \in \mathcal{N}} x_{s,j} &= 1 \text{ for } s \in \mathcal{N} \\
\sum_{i \in \mathcal{N}} x_{i,j} &\leq 1 \text{ for } j = 1, ..., N; j \neq i \\
\sum_{j \in \mathcal{N}} x_{i,j} &\leq 1 \text{ for } i = 1, ..., N; i \neq j \\
x_{i,j} + x_{j,i} &\leq 1 \text{ for all } (i,j) \in \mathcal{A} \\
\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{i,j} &\leq |\mathcal{S}| - 1 \text{ for } \mathcal{S} \in \mathcal{N}, 2 \leq |\mathcal{S}| \leq N - 1 \\
x_{i,j} &\in \{0, 1\}
\end{aligned}$$

Case 2: Static/Stochastic



- Total time of search: 62.85 minutes
- Optimal path: $\psi = [5, 8, 10, 7, 4, 1, 2, 3, 6, 9]$
- Total reward: $r^* = 6$ from nodes 1, 5, 6, 7, 8, 10

Comparison: Deterministic Results

Table 4. Comparison of solutions to the static/deterministic and dynamic/deterministic examples.

Static		*Dynamic*	
Node	r_j	*Node*	$r_j(\tau)$
5	0	5	0
8	1	8	0
10	1	10	0
7	0	7	0
4	0	5	0
1	1	8	0
2	1	10	0
3	1	4	1
6	1	7	1
9	0	10	1
		8	1
		9	1
Total Time (min)	r^*	*Total Time* (min)	$r^*(\tau^*)$
62.85	6	81.72	5

Comparison: Deterministic Results

- Static/deterministic vapor concentration case
  - Search each node exactly once
  - Less amount of time

- Dynamic/deterministic vapor concentration case
  - Searches only critical nodes
  - Utilizes time allotted
  - Total reward value accounts for dynamic nature of concentrations

Comparison: Deterministic Results

Main result of comparisons: Incorporating temporal evolution reduces risk of overestimating/underestimating the number of areas safely operating without protective gear.

- Solution 1: Static
  - 60% of the areas are determined to be safe
  - 33% of those will become unsafe at later times
- Solution 2: Dynamic
  - 50% of areas are determined to be safe
  - 3 of these were previously unsafe

Comparison: Stochastic Results

Table 5. Comparison of solutions to the static/stochastic and dynamic/stochastic examples.

Static		*Dynamic*	
Node	r_j	*Node*	$r_j(\tau)$
5	1	5	0
8	1	1	0
10	1	5	0
7	1	7	1
4	0	10	1
1	1	8	1
2	0	6	1
3	0	3	1
6	1	5	0
9	0	1	0
		4	0
		10	1
Total Time (min)	r^*	*Total Time* (min)	$r^*(\tau^*)$
62.85	6	88.61	5

Comparison: Stochastic Cases

- Static/stochastic vapor concentration case
  - Search each node exactly once
  - Less amount of time
  - Reward based on *expected* vapor concentrations
- Dynamic/stochastic vapor concentration case
  - Search is driven by probability a node will be less than the threshold v^*
  - Rewards determined from *expected* values and rate parameters are time-dependent

Comparison: Stochastic Cases

Main result: Reduce risk of overestimating/underestimating safe areas in dynamic case. Stochastic elements account for randomness of the real problem.

- Solution 1:
  - 60% of areas are determined to be safe
  - Following this path declares safe areas prematurely
  - 2 of the areas would likely not be safe at later times

Comparison: Stochastic Cases

- Solution 2:
  - 50% of areas are determined to be safe
  - 1 of the unsafe nodes in the previous case becomes safe at a later time